Valuations of Warrants by Multiple Tree Model

2019/12/30

Outline

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- 關於warrants
- Valuations of warrants
- Build asset tree
- Backward Induction
    - Maturity
    - Prior to maturity
    - Issuance

為何會有這個報告

- 我目前的文章是介紹一個評價可轉債(CB)的 Model，其特色為可以捕捉CB部分轉換的現象
- 早期文獻認為CB的最佳轉換時機為到期日前一日與股利發放日前一日，且最佳轉換比例為100%
- 但是觀察實際資料可以觀察到「到期前部分轉換」的現象
- 為了介紹這個Model需要一個額外的例子，藉由評價Warrant來完善這點

為何會有這個報告

- Constantinides(1984) 探討了CB在到期日前的逐步部分現象，將可轉債視為一張普通債與許多Warrants的集合，以經濟學的完全競爭理論，解釋部分轉換的現象。

- Bühler, Wolfgang and Koziol, Christian(2002) 考慮獨占情況下CB會有逐步轉換的現象，在同時考慮稅盾與破產成本後，獨占的CB持有者可以利用最佳轉換比例以最大化CB的價值。

關於warrants

	Equity warrants	Derivative warrants
德國交易所	Traditional warrants	Naked warrants
香港交易所	股本認售權證 (Equity warrants)	衍生權證 (Derivative warrants)
台灣櫃買	從缺	權證
發行方	公司	交易所、券商
履約支應方式	發行新股	現金結算

- 歐洲交易所（德國）將warrants視為與有關債券之金融商品，但是在美洲（NYSE, NASDAQ, Canadian）則是在股票交易所交易

關於warrants

- 傳統的warrants評價由warrants持有方進行評價：

  W_T is the value of a warrant at maturity

  N is the number of common shares outstanding

  M is the number of warrants

  E_t is equity value at time t

  X is exercise price for 1% share of stock

  C_P is convert proportion (a warrant exercise convert to C_P share)

  $y\,\%$ is optimal cumulative exercise percentage at time t

  $x\,\%$ is previous cumulative exercise percentage at time t

$$W_T = Max(\frac{E_T + M(y\% - x\%)X}{N + M(y\% - x\%)C_p} - X, 0)$$

Valuations of warrants

- 我們的warrants評價由公司方進行評價：

  WV_t is the value of warrants at time t

  V_t is firm value at time t

  SB_T is the repayment of the straight bond at maturity

  N_B is the number of straight bond

  F_B is the face value of straight bond

  C_B is the coupon rate of straight bond

  B_c is the bankruptcy cost

  tax is the tax rate

  r_f is the risk-free interest rate

Valuations of warrants

- 我們的warrants評價由公司方進行評價：

- 假定warrants發行由一人持有，此種情況與經濟學中的獨占市場相同，warrants履約時持有人成為新股東，公司獲得注資，但由於股價稀釋效應，若一次履約太多可能會減損warrants履約時的價值，此人可在不同時間點選擇最佳履約比例。

- 假定warrants發行由許多人持有，此種情況與經濟學中的完全競爭市場相同，warrants持有者履約時無法對股價有稀釋效應，只要履約的價值大於持續持有的價值就會履約。

Build asset tree

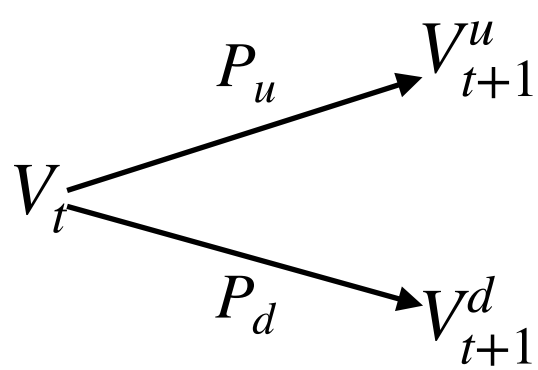
- 以Brodie and Kaya二元樹模型模擬未履約Firm Value，設定公司資產之隨機過程為：

$$dlnV(t) = (r_f - q - \frac{\sigma_v^2}{2})dt + \sigma_v dZ_v$$

$$u = e^{\sigma_v\sqrt{\Delta t}} \qquad r = r_f - q$$

$$d = e^{-\sigma_v\sqrt{\Delta t}} \qquad P_u = \frac{e^{r\Delta t} - d}{u - d}$$

$$P_d = 1 - P_u$$



$$\delta_t = V_t e^{q\Delta t} - V_t$$

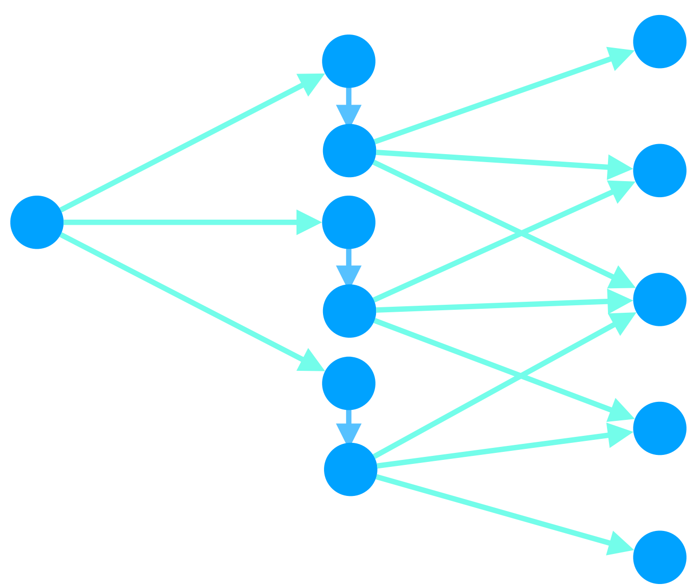
$$P_u(V_{t+1}^u + \delta_{t+1}^u) + P_d(V_{t+1}^d + \delta_{t+1}^d) = V_t e^{r_f\Delta t} \qquad \bar{\delta}_T = V_t e^{(q-B_c)\Delta t} - V_t$$

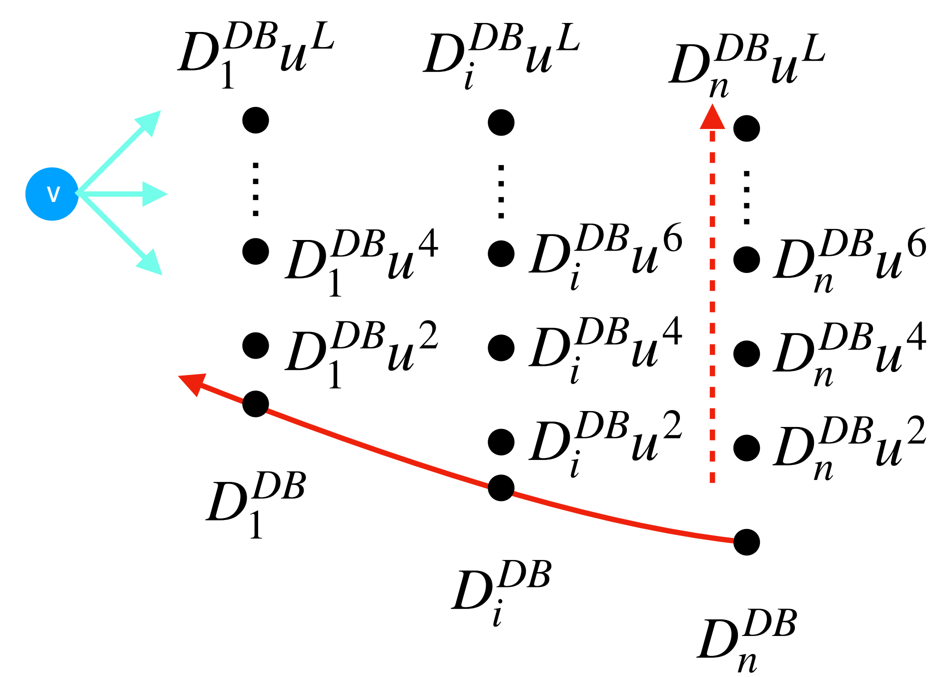
- 設定資本支出為 δ ，資本支出率為 q，資本支出優先支付給普通債的債息，若有多的部分則為股利，並設定公司無力償付普通債債息時，發行新股籌措資金，但會造成在外流通股數增加。
- 假設公司在外流通股數為100，可將轉換比例**Cp**%解釋成一份(1%) warrant可以轉換**Cp**份(**Cp**%)的股票，將原本的股價解釋為1%股權的價值，這樣就可以避免因為發行新股造成轉換比例變動的問題

Build asset tree

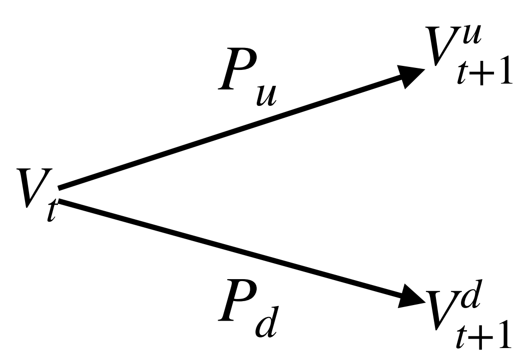
- 外生門檻：$D_i^{DB} = D_{t+1}^{DB} e^{-r_f \Delta t} + C_B(1 - tax) \quad t = i$

  $D_n^{DB} = Debt + C_B(1 - tax) \quad t = n$





- 內生門檻： $E_t < 0$，E_t為未來所有E_{t+i}的期望值折現



Build asset tree

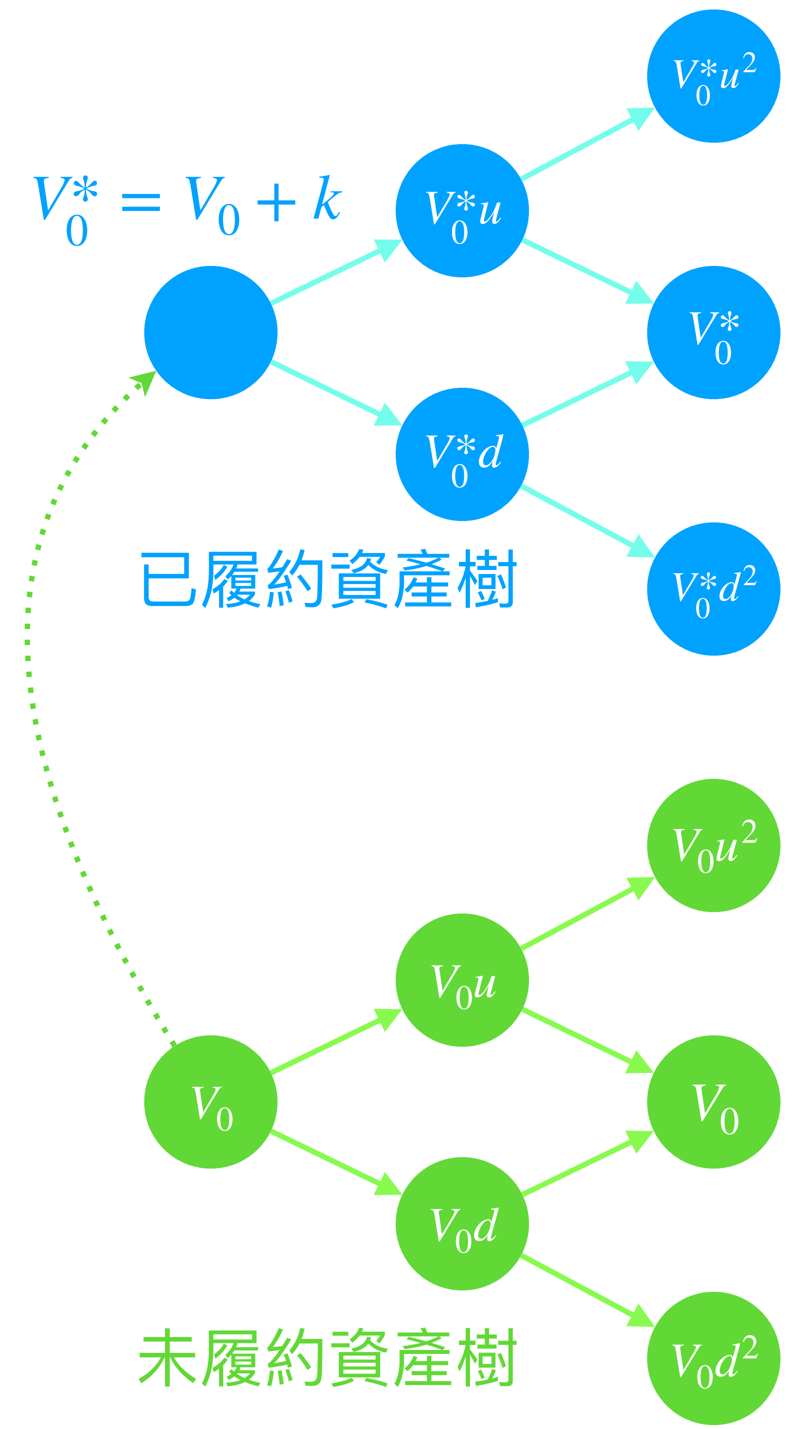
- 因為Warrant履約對公司來說有注資效果，所以資產樹需要多層次的樹狀結構，與CB不同，CB轉換僅改變資本結構沒有注資效果，不需要多層次的樹狀結構
- Warrant履約注資：$k = M(y\% - x\%)X$

當期累積履約比率 y%

前期累積履約比率 x%

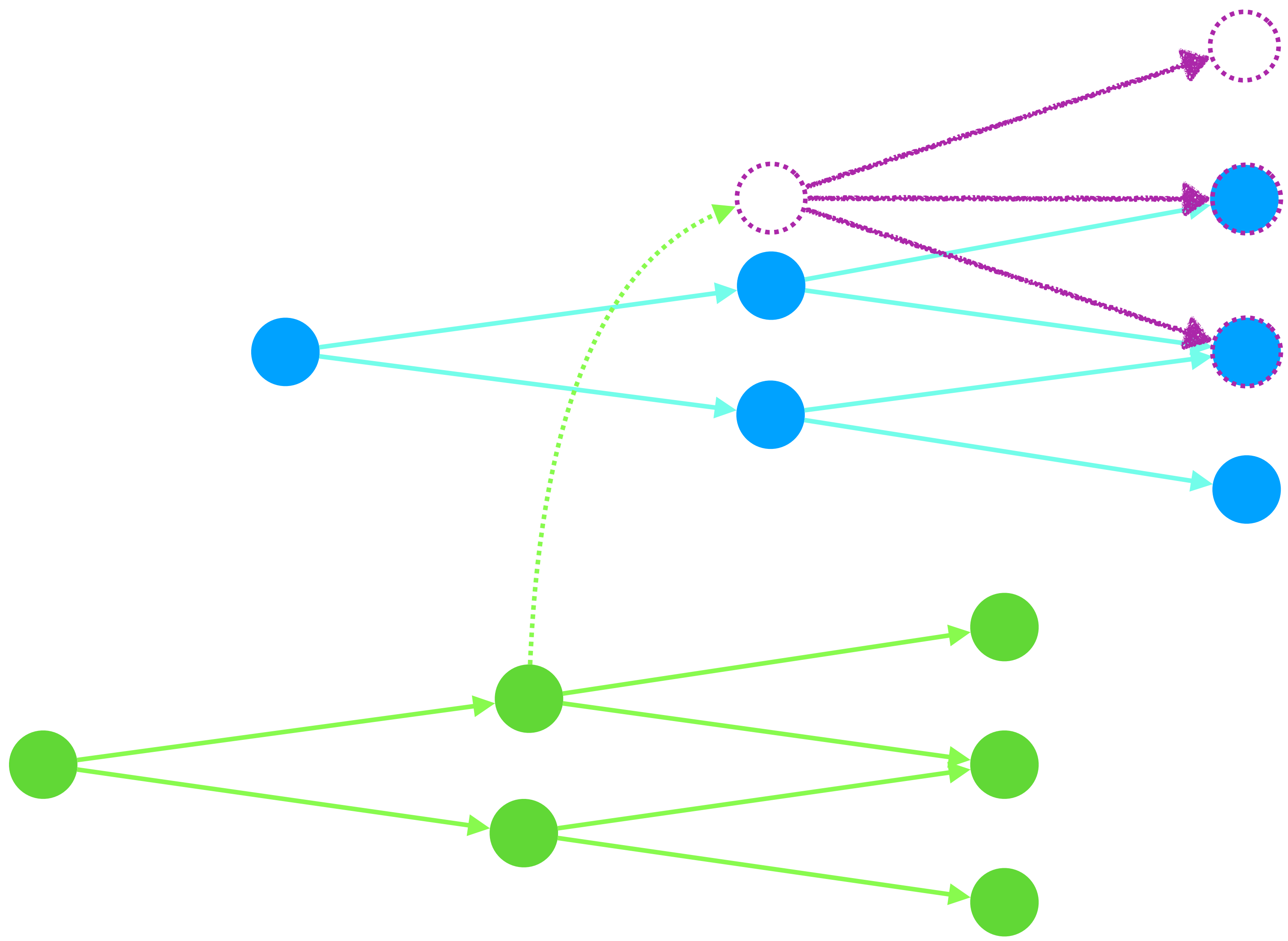
	0%	50%	100%
0%			
50%	-		
100%	-	-	

$y\% - x\% = 50\% - 0\%$



Build asset tree

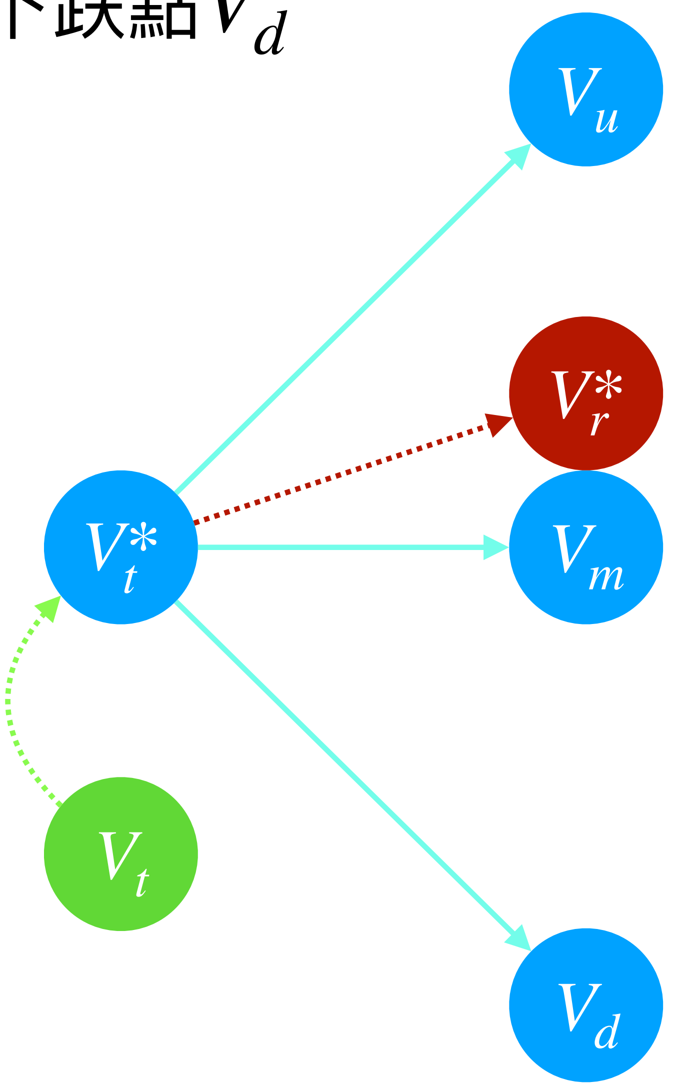
- 但是上方樹的形狀無法直接確定，需要將每個節點的注資都計算後才能確定，除了形狀之外上下界也需要尋找



Build asset tree

- 設V_t^*為履約後注資之公司資產價值且$V_r^* = V_t^* e^{r\Delta t}$
- 以V_r^*最近的網格點為V_m，在分別取上漲點V_u與下跌點V_d

$$\begin{cases} \begin{bmatrix} \tilde{P}_u & \tilde{P}_m & \tilde{P}_d \end{bmatrix} \begin{bmatrix} ln(V_u) - ln(V_r^*) \\ ln(V_m) - ln(V_r^*) \\ ln(V_d) - ln(V_r^*) \end{bmatrix} = 0 \\ \begin{bmatrix} \tilde{P}_u & \tilde{P}_m & \tilde{P}_d \end{bmatrix} \begin{bmatrix} (lnV_u - lnV_r^*)^2 \\ (lnV_m - lnV_r^*)^2 \\ (lnV_d - lnV_r^*)^2 \end{bmatrix} = \sigma_v^2 \Delta t \\ \tilde{P}_u + \tilde{P}_m + \tilde{P}_d = 1 \end{cases}$$

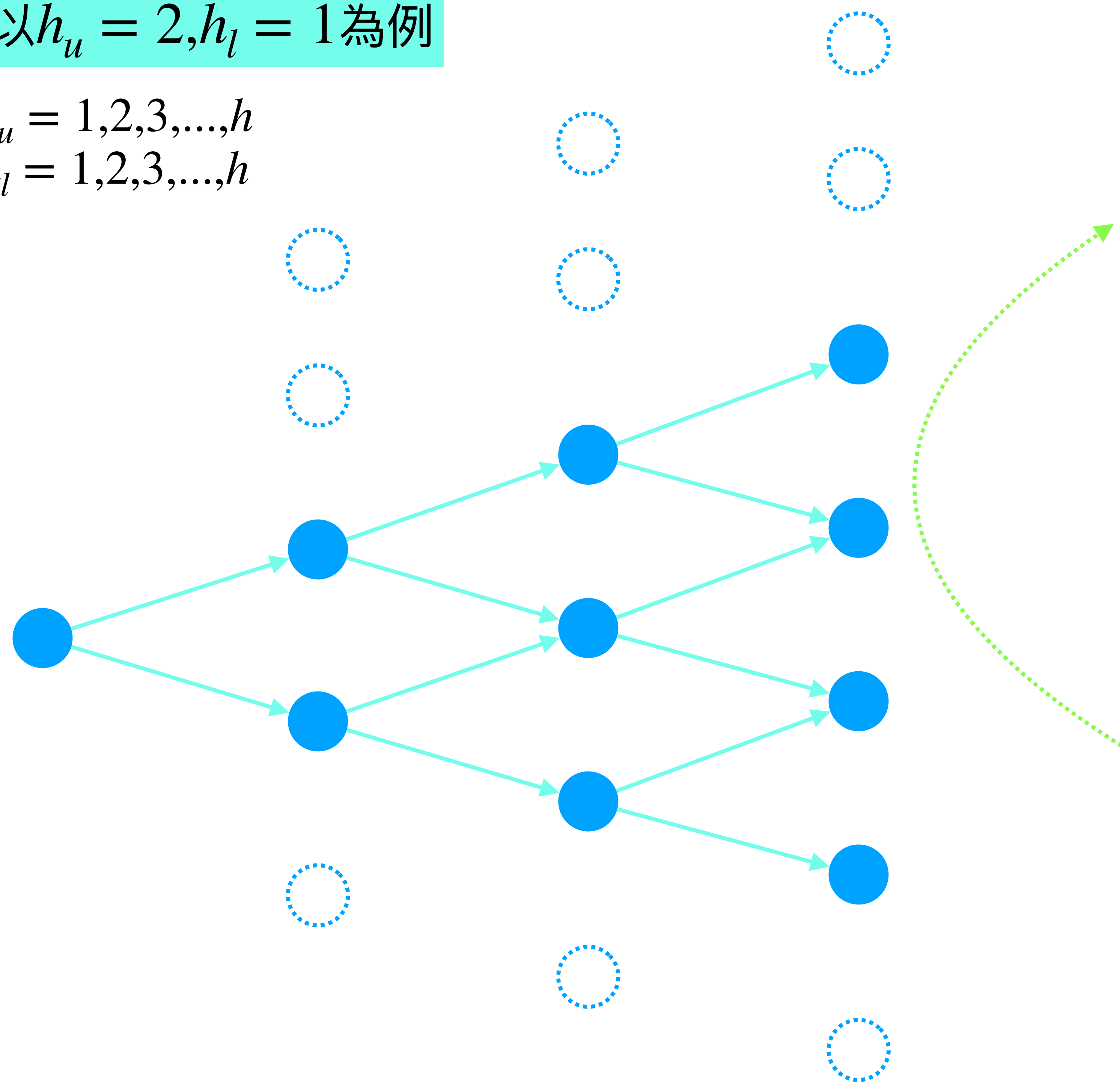


驗算： $\begin{bmatrix} \tilde{P}_u & \tilde{P}_m & \tilde{P}_d \end{bmatrix} \begin{bmatrix} ln(Vu) \\ ln(V_m) \\ ln(V_d) \end{bmatrix} = (\tilde{P}_u + \tilde{P}_m + \tilde{P}_d) ln(V_t^* e^{r\Delta t})$

Build asset tree

以$h_u = 2, h_l = 1$為例

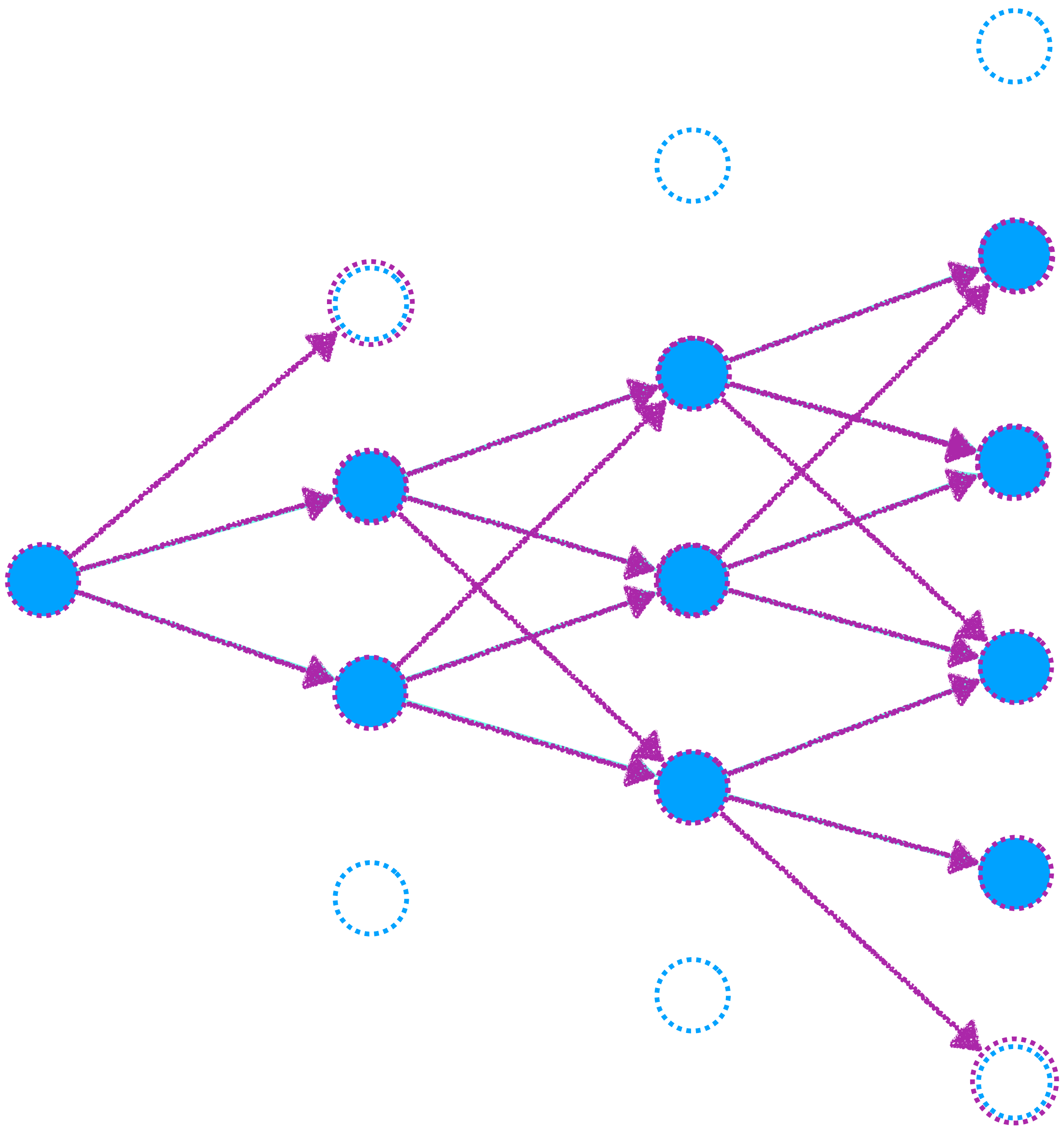
$h_u = 1,2,3,\ldots,h$
$h_l = 1,2,3,\ldots,h$



NaN	$V_0^* u^5$	$V_0^* u^6$	$V_0^* u^7$
NaN	$V_0^* u^3$	$V_0^* u^4$	$V_0^* u^5$
V_0^*	$V_0^* u$	$V_0^* u^2$	$V_0^* u^3$
NaN	$V_0^* d$	V_0^*	$V_0^* u$
NaN	$V_0^* d^3$	$V_0^* d^2$	$V_0^* d$
NaN	NaN	$V_0^* d^4$	$V_0^* d^3$
NaN	NaN	NaN	$V_0^* d^5$

V_0	$V_0 u$	$V_0 u^2$	$V_0 u^3$
NaN	$V_0 d$	V_0	$V_0 u$
NaN	NaN	$V_0 d^2$	$V_0 d$
NaN	NaN	NaN	$V_0 d^3$

Build asset tree

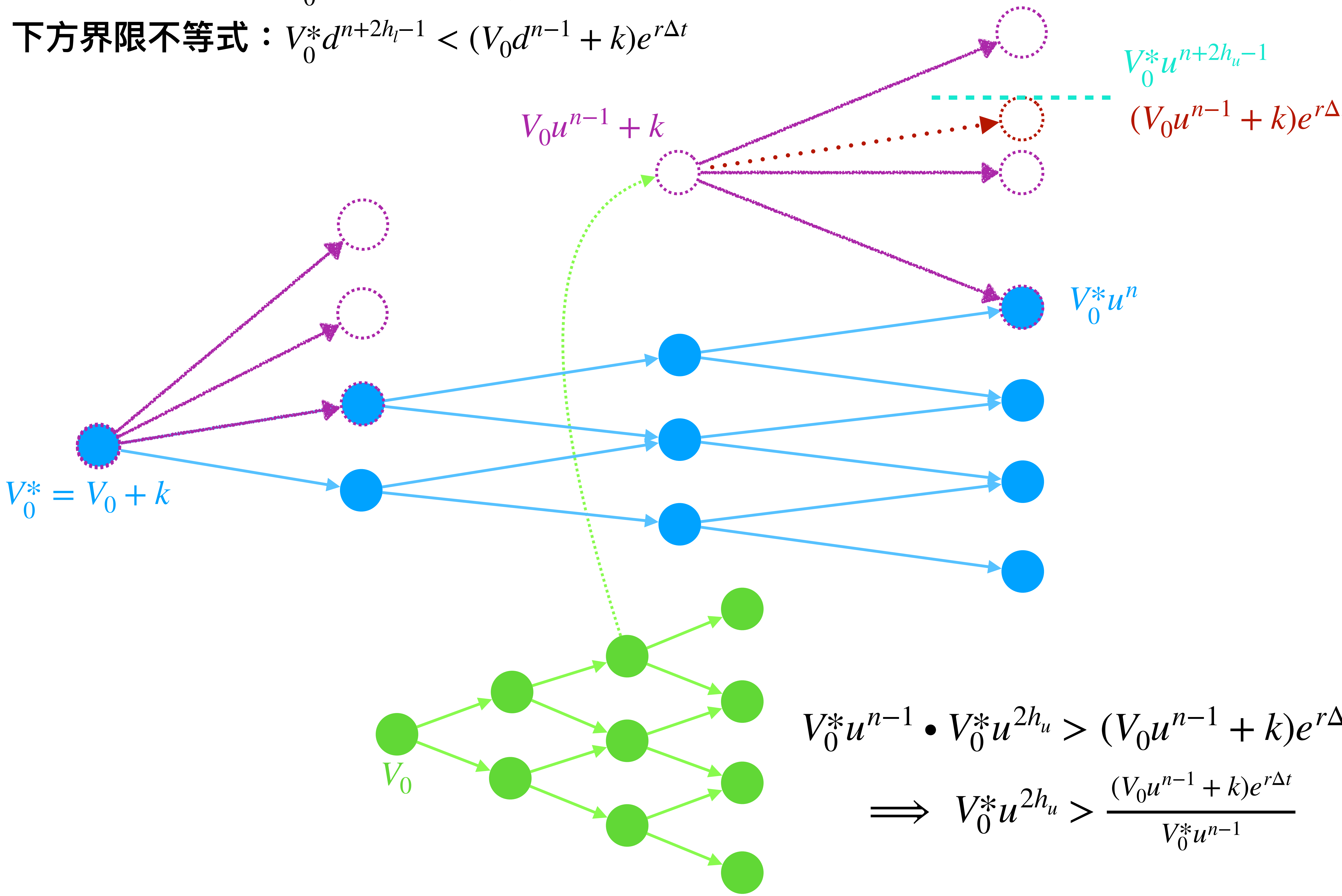


NaN			
NaN			
NaN			
NaN	NaN		
NaN	NaN	NaN	

NaN			
NaN	NaN		
NaN	NaN	NaN	

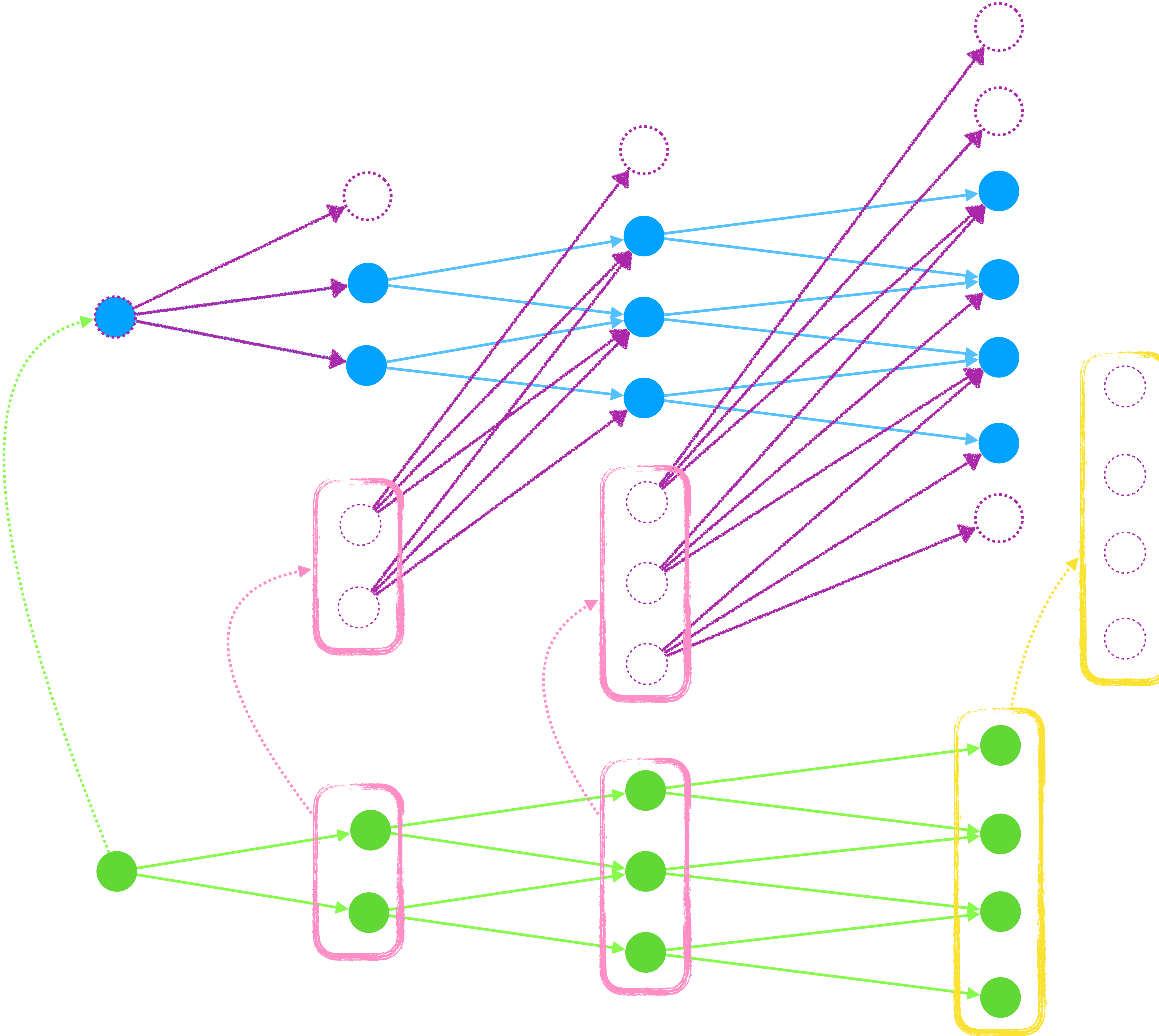
上方界限不等式：$V_0^* u^{n+2h_u-1} > (V_0 u^{n-1} + k)e^{r\Delta t}$

下方界限不等式：$V_0^* d^{n+2h_l-1} < (V_0 d^{n-1} + k)e^{r\Delta t}$



$$V_0^* u^{n-1} \bullet V_0^* u^{2h_u} > (V_0 u^{n-1} + k)e^{r\Delta t}$$

$$\Longrightarrow \quad V_0^* u^{2h_u} > \frac{(V_0 u^{n-1} + k)e^{r\Delta t}}{V_0^* u^{n-1}}$$



Backward Induction

- Step1:先將Warrant履約後的資產樹建出來，全部Backward算出個節點的值
- Step2:
- Step3:
- Step4:
- Step5:

y%

x%

	0%	50%	100%
0%	Step5	Step4	Step2.2
50%	-	Step3	Step2.1
100%	-	-	Step1

Backward Induction

- 最適履約比例：
- 寡佔：何者履約比例能給warrants持有者帶來最大的價值
- 完全競爭：若未來warrants期望值折現小於履約的價值，持有人就會履約（未完成）

$$E(W_{t+i})e^{-r\Delta t} \leq \text{時點t履約warrants的價值}$$

WV$_t$			
x% \ y%	0%	50%	100%
0%			
50%	-		
100%	-	-	

不同市場情況的
履約策略

完全競爭

寡佔

Backward Induction

Step1:僅計算後一期已履約至前一期已履約的Backward

A1

	0%	50%	100%
0%			
50%	-		
100%	-	-	

B1

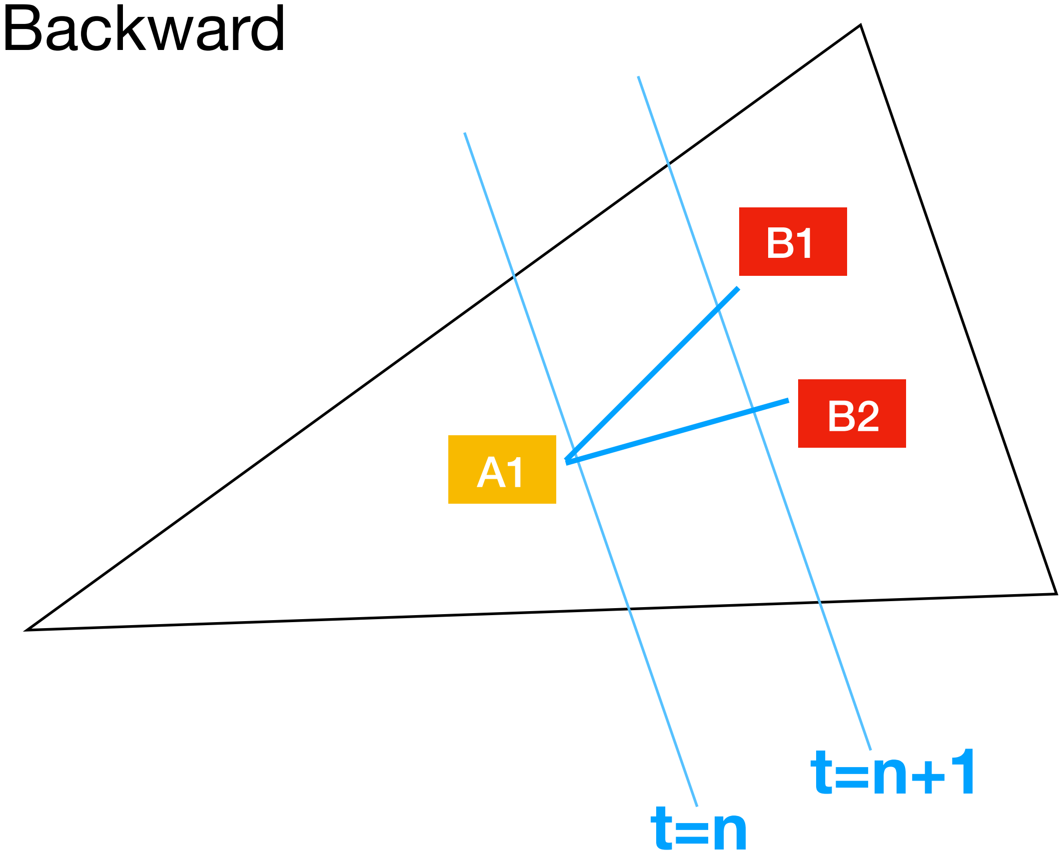
	0%	50%	100%
0%			
50%	-		
100%	-	-	

B2

	0%	50%	100%
0%			
50%	-		
100%	-	-	

P_u

P_d

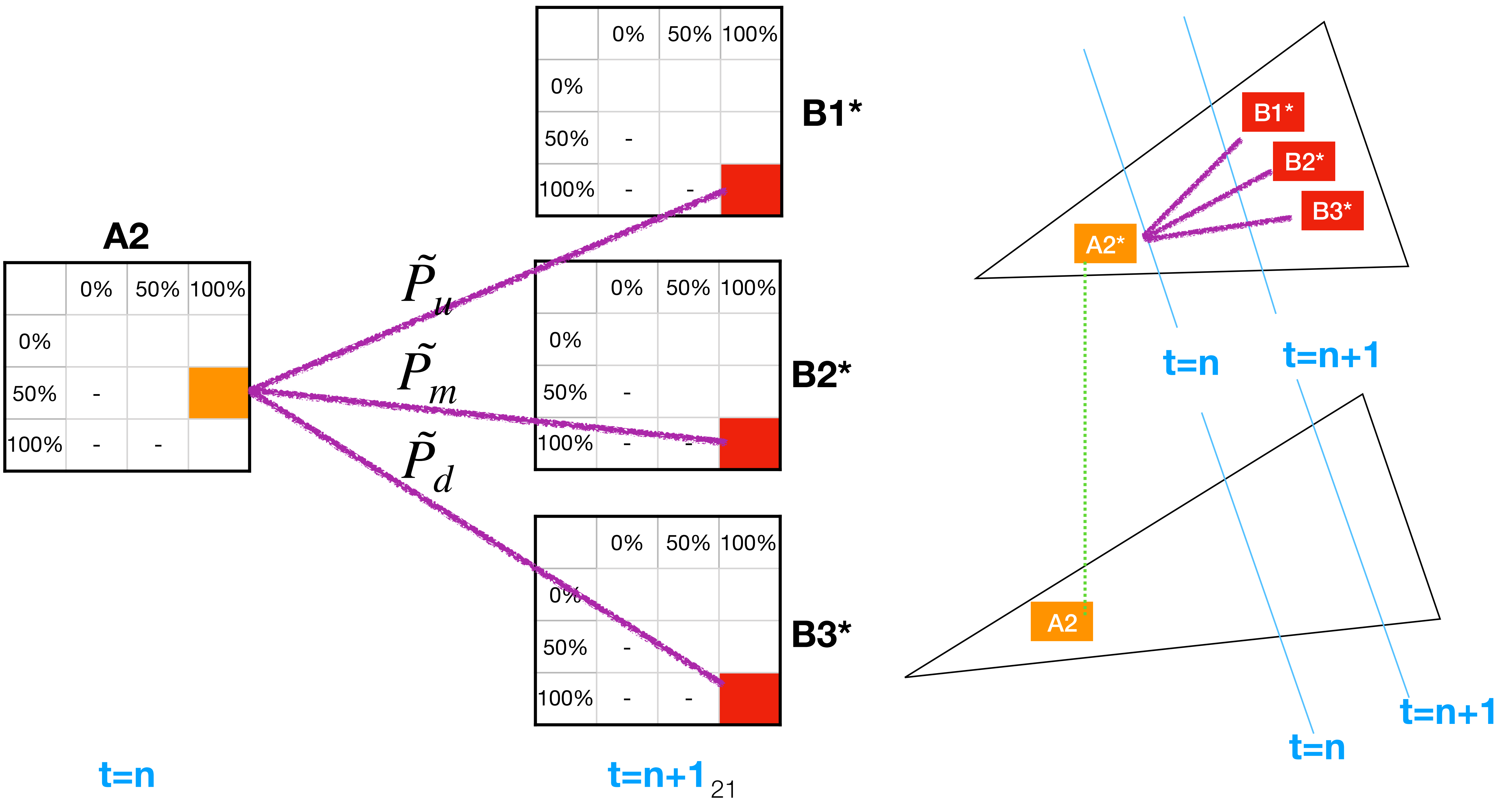


t=n

t=n+1

Backward Induction

Step2:



Backward Induction

Step3:

B1*與B2*的累積履約比例是
由最適履約比例決定

A2

	0%	50%	100%
0%			
50%	-		
100%	-	-	

P_u

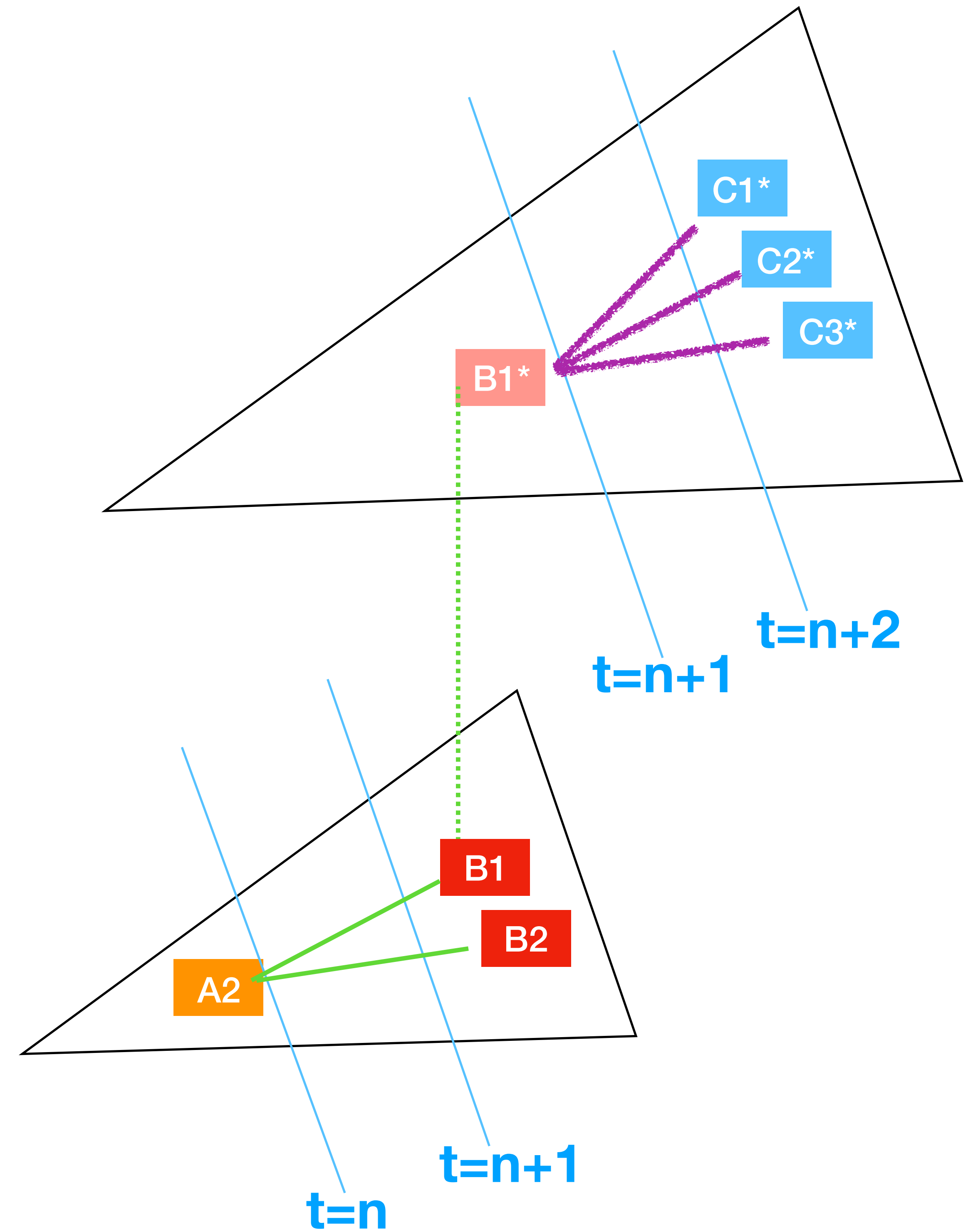
B1*

	0%	50%	100%
0%			
50%	-		
100%	-	-	

P_d

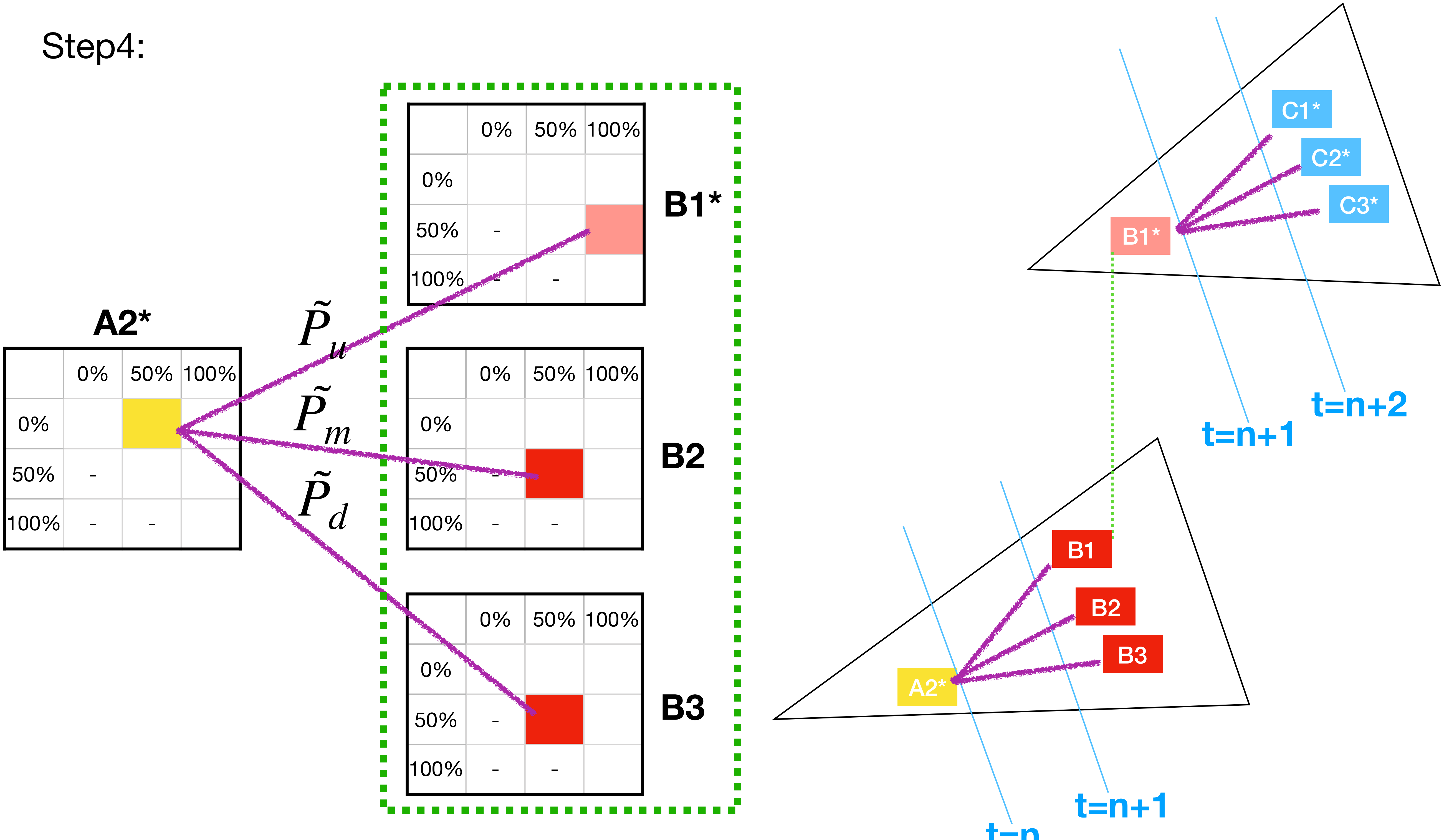
B2

	0%	50%	100%
0%			
50%	-		
100%	-	-	



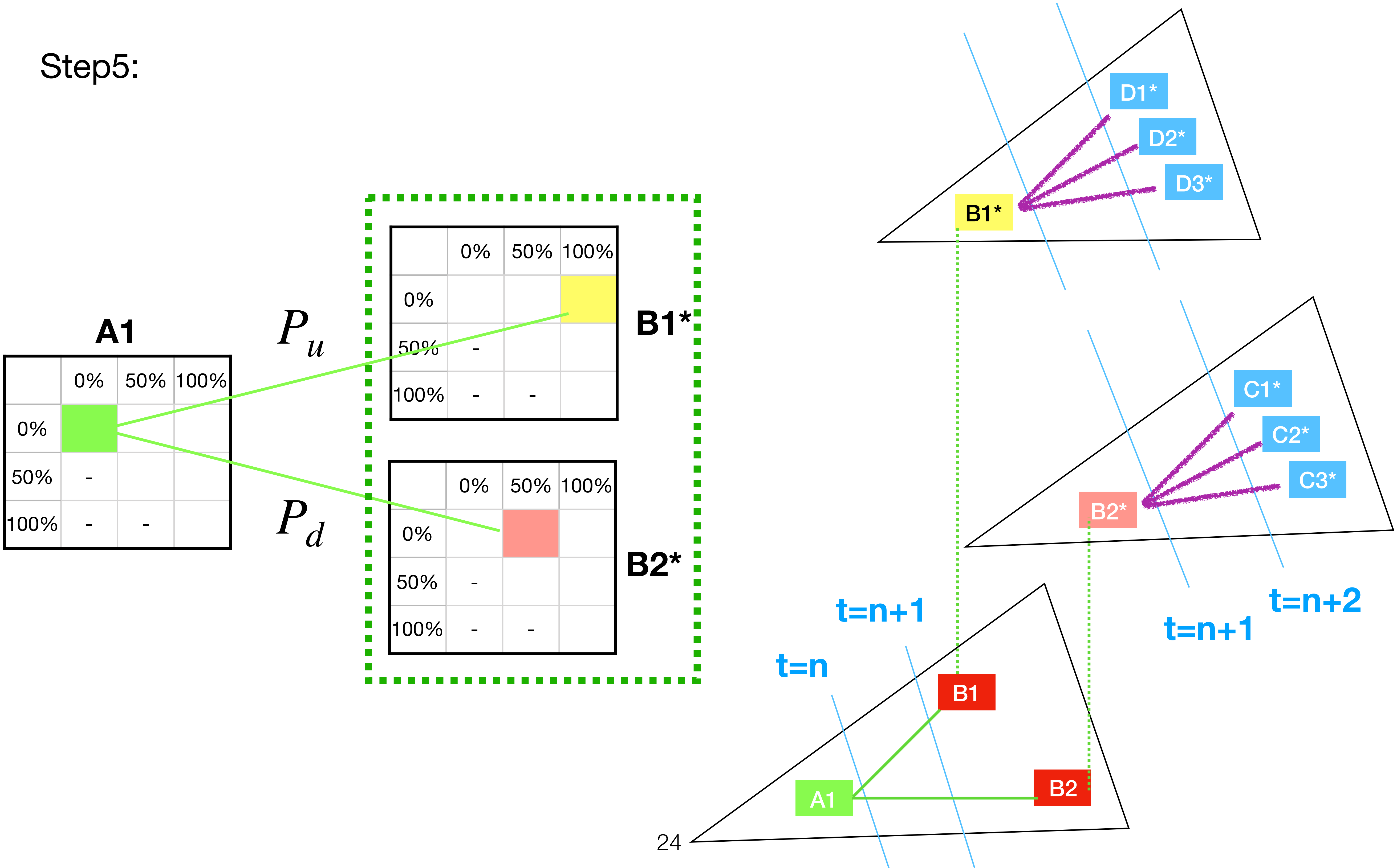
Backward Induction

Step4:



Backward Induction

Step5:



At maturity

Equity：$E_T = V_T + \delta - N_B F_B (1 + (1 - tax) C_B \Delta t)$

At maturity and firm survive：$E_T > 0$

Stock Price：$SP_T = \dfrac{E_T}{N + My\,\%\,C_p}$

Warrant Value：$WV_T = Max(SP_T - X, 0) \bullet M(y\% - x\%) C_p$

Staright Bond：$SB_T = N_B F_B + N_B F_B C_B \Delta t$

At maturity and firm default：$E_T \leq 0$ $\quad SP_T = 0, \quad WV_T = 0$

Staright Bond：$SB_T = min(N_B F_B (1 + C_B \Delta t), (1 - Bc)(V_T + \delta))$

驗算式（未破產）：$V_T + N_B F_B C_B tax \Delta t + \delta + M(y\% - x\%)X = E_T + SB_T$

驗算式（破產）：$V_T - B_c(V_T + \delta) + \delta + M(y\% - x\%)X = E_T + SB_T$

Prior to maturity

Equity：$E_t = (N + My\,\%\,C_p) \bullet PV_t^{stock} + \delta - N_B F_B C_B \Delta t(1 - tax)$

Present value of stock：$PV_t^{stock} = (SP_{t+1}^u \bullet \tilde{P}_u + SP_{t+1}^m \bullet \tilde{P}_m + SP_{t+1}^d \bullet \tilde{P}_d)e^{-r\Delta t}$

Prior to maturity and firm survive：$E_t > 0$

Stock Price：$SP_t = \dfrac{E_t}{N + My\,\%\,C_p}$

Warrant Value：$WV_t = (WV_{t+1}^u \bullet \tilde{P}_u + WV_{t+1}^m \bullet \tilde{P}_m + WV_{t+1}^d \bullet \tilde{P}_d)e^{-r\Delta t} \bullet M(1 - y\%)C_p + \ldots$
$\ldots + Max(SP_t - X, 0) \bullet M(y\% - x\%)C_p$

Staright Bond：$SB_t = (SB_{t+1}^u \bullet \tilde{P}_u + SB_{t+1}^m \bullet \tilde{P}_m + SB_{t+1}^d \bullet \tilde{P}_d)e^{-r\Delta t} + N_B F_B C_B \Delta t$

Prior to maturity and firm default：$E_t \le 0 \qquad SP_t = 0, \quad WV_t = 0$

Staright Bond：$SB_t = min(CFofSB_t + N_B F_B C_B \Delta t), (1 - Bc)(V_t + \delta))$

Cash Flow of Staright Bond discount : $CFofSB$

驗算式：$V_t + \delta + M(y\% - x\%)X = E_t + SB_t + WV_t$

Issuance

Equity：$E_0 = (N + My\,\%\,C_p) \bullet PV_0^{stock} + \delta - N_B F_B C_B \Delta t(1 - tax)$

Present value of stock：$PV_0^{stock} = (SP_1^u \bullet \tilde{P}_u + SP_1^m \bullet \tilde{P}_m + SP_1^d \bullet \tilde{P}_d)e^{-r\Delta t}$

Issuance and firm survive：$E_0 > 0$

Stock Price：$SP_0 = \dfrac{E_0}{N + My\,\%\,C_p}$

Warrant Value：$WV_0 = (WV_1^u \bullet \tilde{P}_u + WV_1^m \bullet \tilde{P}_m + WV_1^d \bullet \tilde{P}_d)e^{-r\Delta t} \bullet M(1 - y\%)C_p + \ldots$

$\ldots + Max(SP_0 - X, 0) \bullet M(y\% - x\%)C_p$

Staright Bond：$SB_0 = (SB_1^u \bullet \tilde{P}_u + SB_1^m \bullet \tilde{P}_m + SB_1^d \bullet \tilde{P}_d)e^{-r\Delta t} + N_B F_B C_B \Delta t$

Issuance and firm default： $E_0 \leq 0$

$$SP_0 = 0, \quad WV_0 = 0$$

Staright Bond：$SB_0 = min(CFofSB_0, (1 - Bc)(V_0 + \delta))$

驗算式：$V_0 + \delta + M(y\% - x\%)X = SB_0 + E_0 + WV_0$